On the numerical range of powers of matrices

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Abstract

The numerical range of a matrix $A \in \mathbb{C}^{n \times n}$ is the set defined by

$$W(A) = \{ \langle Ax, x \rangle : x \in \mathbb{C}^n, \| x \| = 1 \}$$

where $\langle x, y \rangle = y^* x$ is the inner product of $x, y \in \mathbb{C}^n$ and $\| x \| = \sqrt{\langle x, x \rangle}$ is the Euclidean norm. One of the basic properties of $W(A)$ is that it contains all eigenvalues of $A$.

We consider the following problem. Suppose the numerical range $W(A^k)$ of all powers $k = 1, 2, \ldots$ of a given matrix $A \in \mathbb{C}^{n \times n}$ is contained in some strip that is not parallel to the real axis. We show that in this case $A$ is power bounded. The extension of this result to linear operators on a Hilbert space is also given.

Keywords

Numerical range, Operator norm, Power bounded operators.

References:

