Invariance of matrix expressions with respect to specific classes of generalized inverses

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Abstract

Several results are known in the literature concerning the invariance of the product $\mathbf{AB}^{(1)}\mathbf{C}$ itself and some expressions involving it with respect to the choice of a generalized inverse $\mathbf{B}^{(1)}$ of $\mathbf{B}$, i.e., with respect to all matrices satisfying $\mathbf{BB}^{(1)}\mathbf{B} = \mathbf{B}$. The collection of other expressions mentioned above contains in particular the range, rank, spectrum, trace, and Frobenius norm of $\mathbf{AB}^{(1)}\mathbf{C}$. The purpose of the present paper is to revisit these results from the viewpoint of weakening the invariance requirement to the subsets of $\{\mathbf{B}^{(1)}\}$ comprising the matrices, which in addition to $\mathbf{BB}^{(1)}\mathbf{B} = \mathbf{B}$ satisfy also further conditions from a definition of the Moore-Penrose inverse of $\mathbf{B}$, i.e., $\mathbf{B}^{(1)}\mathbf{BB}^{(1)} = \mathbf{B}^{(1)}$ and/or $\mathbf{BB}^{(1)} = (\mathbf{BB}^{(1)})^*$ and/or $\mathbf{B}^{(1)}\mathbf{B} = (\mathbf{B}^{(1)}\mathbf{B})^*$.

Keywords

Generalized inverse, Range, Rank, Spectrum, Trace, Frobenius norm.