

On the numerical range of powers of matrices

Iwona Wróbel¹ and Jaroslav Zemánek²

¹Warsaw University of Technology, Warsaw, Poland

²Polish Academy of Sciences, Warsaw, Poland

Abstract

The numerical range of a matrix $A \in \mathbb{C}^{n \times n}$ is the set defined by

$$W(A) = \{\langle Ax, x \rangle : x \in \mathbb{C}^n, \|x\| = 1\}$$

where $\langle x, y \rangle = y^*x$ is the inner product of $x, y \in \mathbb{C}^n$ and $\|x\| = \sqrt{\langle x, x \rangle}$ is the Euclidean norm. One of the basic properties of $W(A)$ is that it contains all eigenvalues of A .

We consider the following problem. Suppose the numerical range $W(A^k)$ of all powers $k = 1, 2, \dots$ of a given matrix $A \in \mathbb{C}^{n \times n}$ is contained in some strip that is not parallel to the real axis. We show that in this case A is power bounded. The extension of this result to linear operators on a Hilbert space is also given.

Keywords

Numerical range, Operator norm, Power bounded operators.

References:

- Finckenstein, K. (1970). Potenzbeschränktheit und Wertebereich einer Matrix. *Numer. Math.* 15, 329–332.
- Gustafson K. E. and D. K. M. Rao (1997), *Numerical range*, New York: Springer-Verlag.
- Horn R. A. and C. R. Johnson (1985), *Matrix analysis*, Cambridge: Cambridge University Press.