On projectors with respect to seminorms

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Abstract

Let A be an $m \times n$ matrix and V be an $m \times m$ nonnegative definite matrix. A square matrix $P_{A:V}$ is called a projector into the column space $\mathcal{R}(A)$ of A with respect to the seminorm $\|\mathbf{z}\|_V$ defined by $\|\mathbf{z}\|_V = (\mathbf{z}'V\mathbf{z})^{1/2}$ if

$P_{A:V}\mathbf{y} \in \mathcal{R}(A)$ and $\|\mathbf{y}-P_{A:V}\mathbf{y}\|_V \leq \|\mathbf{y}-A\mathbf{x}\|_V$ for any $\mathbf{x} \in \mathbb{R}^{n\times 1}$, $\mathbf{y} \in \mathbb{R}^{m\times 1}$.

The projector $P_{A:V}$ is not necessarily idempotent for a given pair of matrices A and V. In this paper, we investigate various properties of $P_{A:V}$ under the three conditions: (i) V is positive definite, (ii) $\mathrm{rank}(VA) = \mathrm{rank}(A)$, (iii) $\mathrm{rank}(VA) < \mathrm{rank}(A)$.

Keywords

Seminorm, Moore-Penrose inverse, weighted Moore-Penrose inverse, projector, matrix equality, rank equality, range equality, commutativity

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