Canonical form of a linear model and its applications

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Abstract

Arbitrary linear model of type $L(A\beta, V)$, $L(A\beta, \sigma V)$ or $N(A\beta, \sigma V)$ for a random sample $X_1, \ldots, X_n$ is considered. It is well known that if the model is *regular* in the sense $R(A) \subseteq R(V)$ then it admits a representation by a sample $Y_1, \ldots, Y_m$ $(m \leq n)$, where $EY_i = \eta_i$ for $i = 1, \ldots, k$ and zero for $i = k+1, \ldots, m$, while $Cov(Y_i, Y_j) = \delta_{ij}$ or $\sigma\delta_{ij}$, respectively. This canonical form, introduced by Kołodziejczyk (1935), was used, among others, by Schéffe (1959) and Lehmann (1959, 1986) for linear estimation and testing linear hypotheses. It appears that this technique may be extended for *arbitrary*, not necessarily regular model and, what is more, it may be applied for quadratic estimation. This enables us to derive many results in a simple way.

References:

Kołodziejczyk, S. (1935). On an important class of statistical hypotheses. *Biometrika 27*, 161-190.

Lehmann, E.L. (1959). *Testing Statistical Hypotheses*. New York: Wiley (2nd ed. 1986).

Schéffe, H. (1959). *Analysis of Variance*. New York: Wiley.