One-sample spatial sign and rank methods

Seija Sirkiä and Hannu Oja

University of Jyväskylä, Jyväskylä, Finland

Abstract

Consider a sample $x_1, \ldots, x_n$ from a p-variate elliptically symmetric distribution with density $f(x; \mu, \Omega) = |\Omega|^{-\frac{1}{2}} g(x^T \Omega^{-1} x)$. We wish to estimate the location vector μ and the scatter matrix Ω or the so called shape matrix $V = \frac{p}{Tr(\Omega)} \Omega$. In this talk several affine equivariant estimates for location and shape are considered. These are based on the concepts of multivariate, or spatial, signs, $S(x_i) = ||x_i||^{-1} x_i$, and ranks, $R(x_i) = ave_{j \neq i} \{S(x_i - x_j)\}$. Hettmansperger and Randles (2002) proposed a special case of M-estimates for location and shape, namely, a combination of the so called transformation-retransformation median (Chakraborty et al., 1998) and Tyler's M-functional (Tyler, 1987). Unfortunately, the simultaneous existence and uniqueness of these estimators have not been proven, although the algorithm seems to work very well in practice. Another possibility for shape estimation is to use pairwise differences of observations and spatial signs of those (Dümbgen, 1998). This Kendall's Tau -type estimate can be shown to exist uniquely, there is a simple algorithm for it and it is highly efficient. Yet another estimate can be based on spatial ranks, resulting in a Spearman's Rho -type estimate, but proofs for its existence and uniqueness seem to be much more difficult and have not yet been completely found.

Keywords

Multivariate location and shape, Multivariate signs and ranks, Affine equivariance

References:

Hettmansperger, T.P. and R.H. Randles (2002). A practical affine equivariant multivariate median. *Biometrika 89*, 851–860.

Chakraborty, B., P. Chaudhuri , and H. Oja (1998). Operating transformation retransformation on spatial median and angle test. *Statist. Sinica 8*, 767-784.

Tyler D. E. (1987). A distribution-free M-estimator of multivariate scatter. *Ann. Statist. 15*, 234–251.

Dümbgen L. (1998). On Tyler's M-functional of scatter in high-dimension. *Ann. Inst. Statist. Math. 50*, 471-491.