Some results on patterned matrices

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Abstract

We are going to consider patterned matrices as subsets of matrix elements without tying the notion of patterned matrix to any specific relation among the elements of the matrix. A patterned matrix $A(K)$ is a matrix where any element or a certain part of the original matrix, defined by an index-set $K$, has been excluded from $A$, i.e. a certain pattern has been “cut out” from the original matrix. The major part of the applications of the approach concerns symmetric, skew-symmetric, diagonal, Toeplitz, triangular, etc. matrices.

Let $A$ be an $p \times q$-matrix and $K$ a set of pairs of indices:

$$K = \{(i, j) : i \in I_K, j \in J_K; I_K \subset \{1, \ldots, p\}; J_K \subset \{1, \ldots, q\}\}.$$ 

We call $A(K)$ a patterned matrix and the set $K$ a pattern of the $p \times q$-matrix, if $A(K)$ consists of elements $a_{ij}$ of $A$ where $(i, j) \in K$.

The notation $A(K)$ does not represent a matrix in a strict sense since it is not a rectangle of elements. One should just regard $A(K)$ as a convenient notion for a specific collection of elements. When the elements of $A(K)$ are collected into one column by columns of $A$ in a natural order, we get an $r$-vector, where $r$ is the number of pairs in $K$. Let us denote this vector by $\text{vec}A(K)$. Clearly, there exists always a matrix which transforms $\text{vec}A$ into $\text{vec}A(K)$.

We are going to apply a vector space approach and also define several useful matrices in order to present a systematic treatment of patterned matrices. The results turn out to be useful when we are interested in finding Jacobians or want to derive moments of higher order.