Some results on patterned matrices

Dietrich von Rosen

Swedish University of Agricultural Sciences, Uppsala, Sweden

Abstract

We are going to consider patterned matrices as subsets of matrix elements without tieing the notion of patterned matrix to any specific relation among the elements of the matrix. A patterned matrix $\mathbf{A}(K)$ is a matrix where any element or a certain part of the original matrix, defined by an index-set K, has been excluded from $\mathbf{A}$, i.e. a certain pattern has been "cut out" from the original matrix. The major part of the applications of the approach concerns symmetric, skew-symmetric, diagonal, Toeplitz, triangular, etc. matrices.

Let $\mathbf{A}$ be an $p \times q-$matrix and K a set of pairs of indices:

$$K = \{(i,j) : i \in I_K, j \in J_K; I_K \subset \{1,\dots,p\}; J_K \subset \{1,\dots,q\}\}.$$

We call $\mathbf{A}(K)$ a patterned matrix and the set K a pattern of the $p \times q-$matrix, if $\mathbf{A}(K)$ consists of elements a_{ij} of $\mathbf{A}$ where $(i,j) \in K$.

The notation $\mathbf{A}(K)$ does not represent a matrix in a strict sense since it is not a rectangle of elements. One should just regard $\mathbf{A}(K)$ as a convenient notion for a specific collection of elements. When the elements of $\mathbf{A}(K)$ are collected into one column by columns of $\mathbf{A}$ in a natural order, we get an r-vector, where r is the number of pairs in K. Let us denote this vector by $\text{vec}\mathbf{A}(K)$. Clearly, there exists always a matrix which transforms $\text{vec}\mathbf{A}$ into $\text{vec}\mathbf{A}(K)$.

We are going to apply a vector space approach and also define several useful matrices in order to present a systematic treatment of patterned matrices. The results turn out to be useful when we are interested in finding Jacobians or want to derive moments of higher order.