On common divisors of matrices over principal ideal domain

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Abstract

Let R be a principal ideal ring with the identity $e \neq 0$ (Newman, 1972) and R_n the ring of $n \times n$ matrices over R. It is said that the matrices $B, C \in R_n$ have a common left divisor if $B = DB_1,\ C = DC_1$, where $D \in R_n$, $\mathbf{det}D = d \neq 0$ and $D \notin GL(n, R)$.

The problem of common left divisors of matrices over a principal ideal domain is investigated. Necessary and with certain restrictions sufficient conditions are established for existence of a common divisor $D \in R_n$ with a prescribed $\mathbf{det}D = d$ of matrices $B \in R_n$ and $C \in R_n$. In the case, when the desired divisor exists, the method of it constructing is specified. The results are true for elementary divisors rings.

Notation: $B, C \in R_n$; d_A^k – the greatest common divisor of the minors of order k, $1 \leq k \leq n$, of matrix $A = \| \ B \quad C \ \|$.

Proposition. Let $\mathbf{rank}A \geq n - 1$ and d_A^n admits representation in the form $d_A^n = dg$, where $R \ni d \neq 0$ and d is not unit. If $(d, g, d_A^{n-1}) = e$, then for the matrices B and C there exists a left common divisor $D \in R_n$ with given $\mathbf{det}D = d$. If the matrices B and C admit another representations $B = D_1B_2$, $C = D_1C_2$ such that $D_1 \in R_n$ and $\mathbf{det}D_1 = d$, then $D = D_1W$, where $W \in GL(n, R)$. $1 \leq k \leq n$.

Keywords

Matrix, Common divisor.

References:

Newman, M. (1972). *Integral matrices.* New York: Acad. Press.