Unitary invariant random Hermitian matrices and complex elliptical distributions

Esa Ollila and Visa Koivunen

Helsinki University of Technology, Finland

Abstract

Complex random vectors and complex random hermitian matrices which are invariant in distribution under certain transformations by the group of unitary matrices are studied. It is shown that unitary invariance implies a certain structure on their covariance matrix and the pseudo-covariance matrix. (For a zero mean complex random vector $\mathbf{Z}$, the covariance matrix and the pseudo-covariance matrix are defined as $E(\mathbf{Z}\mathbf{Z}^H)$ and $E(\mathbf{Z}\mathbf{Z}^T)$, respectively, where the superscipts T and H denote transpose and conjugate transpose.) This result is then used in the derivation of the finite sample and asymptotic covariance matrix and the pseudo-covariance matrix of *any* affine equivariant estimates of location vector and scatter matrix when sampling from Complex Elliptically Symmetric (CES) distributions of Krishnaiah and Lin (1986). As an example we consider the sample mean and the sample covariance matrix (SCM) in detail. It is well known that when sampling from real elliptically symmetric distributions, the asymptotic covariances between the elements of the SCM can be expressed as a function of the kurtosis and the underlying true covariance matrix, or, as a function of multivariate cumulants (Muirhead, 1982). We show that this is the case also for the covariances and pseudo-covariances of the SCM when sampling from CES distributions. To accomplish this we need to define complex kurtosis and complex multivariate cumulants for complex random variables, and in particular, calculate these for CES distributions.

Keywords

Complex random vectors and matrices, Covariance matrix, Pseudo-covariance matrix, Complex multivariate cumulants, Kurtosis.

References:

Muirhead, R. J. (1982). *Aspects of Multivariate Statistica Theory.* New York: Wiley.

Krishnaiah, P. R. and J. Lin (1986). Complex elliptically symmetric distributions. *Comm. Statist. - Th. and Meth. 15*, 3693–3718.