On the structure of a class of normal decomposition systems

Marek Niezgoda

Agricultural University, Lublin, Poland

Abstract

A normal decomposition system (NDS) is connected with a decomposition statement for vectors of a linear space and with an inequality related to the decomposition. A typical example is the Singular Value Decomposition for matrices provided with the trace inequality of von Neumann. In the paper, we study the problem of generating such systems. The structure of a certain class of NDS is given. As a corollary, we show that the SVD and von Neumann inequality are implied by analogous results of Miranda ad Thompson related to the special orthogonal group.

Keywords:

Normal decomposition system, Eaton triple, G-majorization, Group induced cone ordering, Finite reflection group, Singular value, Eigenvalue.

References:

Eaton, M. L. (1984). On group induced orderings, monotone functions, and convolution theorems. In: Y. L. Tong (Ed.), *Inequalities in Statistics and Probability* (pp. 13–25). IMS Lecture Notes – Monograph Series Vol. 5.

Eaton, M. L., and M. D. Perlman (1977). Reflection groups, generalized Schur functions, and the geometry of majorization. *Ann. Probab. 5*, 829–860.

Lewis, A. S. (1996). Group invariance and convex matrix analysis. *SIAM J. Matrix Anal. Appl. 17*, 927–949.

Marshall, A.W. and I. Olkin (1979). *Inequalities: Theory of Majorization and its Applications*. New York: Academic Press.

Miranda, H. F., and R. C. Thompson (1993). A trace inequality with a subtracted term. *Linear Algebra Appl. 185*, 165–172.

Niezgoda, M. (1998). Group majorization and Schur type inequalities. *Linear Algebra Appl. 268*, 9–30.

Niezgoda, M. (2002). On the structure of a class of Eaton triples. *Forum Math. 14*, 405–411.