Statistical analysis of normal orthogonal models with emphasis on their algebraic structure in view of obtaining effcient statistics for inference

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Abstract

The algebraic structure of normal orthogonal models is presented using Jordan algebras. Such presentation gives both sufficient complete statistics and pivot variables. From the first minimum variance unbiased estimates may be obtained while the second, through the use of the Caratheodory theorem, induces probability measures in the parameter spaces. These will be *a-posteriori* measures since they will depend on the values of sufficient complete statistics, despite no *a-priori* distribution having been assumed. The heavy computations required when dealing with the induced measures may be replaced by the use of Monte Carlo methods validated by the Glivenko-Cantelli theorem and related results.

Moreover, the use of Jordan algebras to describe the structure of the models leads to broader classes of models than those obtained starting with the factors, either fixed effect or random-effect factors, that we consider. Thus, this model formulation leads to a more robust inference. Besides this, a cleaner insight into certain results, such as negative variance components estimators, may be achieved. We point out that factor formulation of the models may be obtained from the algebraic formulation imposing restrictions to the parameters. These restrictions can be tested, thus leading to an integrated inference of both model formulations that may lead to the trimming of non significant factors.

The general results will be applied to models such as those having balanced cross-nesting.

Keywords

Jordan algebras, Mixed models, Algebraic formulation, Factor based formulation, Inductive pivot variables, Duality.

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