

A new rank revealing tri-orthogonalization algorithm and its applications

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Abstract

In this paper we discuss some new numerical methods that are suited for regularization of Linear Least Squares (*LS*) problems with a numerically *rank-deficient* coefficient matrix A . Such problems arise frequently, when we consider inverse problems that need to be solved numerically. The main feature of these problems is that the matrix A in the corresponding overdetermined system of linear equations $Ax = b$ is having a cluster of small singular values, and there is a well determined gap between its large and small singular values.

Usually singular value decomposition of A (*SVD*) is used for that regularization. When A is large and sparse its *SVD* is not very useful, so we propose to apply a new triorthogonalization algorithm followed by the recent versions of Lanczos iterative bidiagonalization with selective reorthogonalization to economize the process of searching for solution of the problem considered. Some numerical results are presented.

Keywords

SVD decomposition, One-sided Householder, Linear Least Squares, Regularization, Divide and Conquer.

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