

Criteria for the comparison of discrete-time Markov chains

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Abstract

In this paper, we develop an approach to compare two discrete-time Markov chains which are not assumed to have the same state space. To do this, we introduce a binary relation between probability vectors or marginal laws which is not a partial order in general. This binary relation generalizes the notion of stochastic ordering for discrete random variables (Muller and Stoyan 2002). We obtain geometric criteria for the comparisons of discrete-time Markov chains. These criteria have the form of inclusion of polyhedral sets. Then, an algebraic form of the previous criteria may be derived from Haar's lemma (Haar 1918). In this context, the so-called concept of positive invariance is in force.

Our result allow us to reexamine some previous works on stochastic comparison of Markov chains. Moreover, we also discuss the comparison of Hidden Markov chains.

Keywords Markov chain, Polyhedral set, Set invariance, Hidden Markov chain

References:

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