

Invariance of matrix expressions with respect to specific classes of generalized inverses

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Abstract

Several results are known in the literature concerning the invariance of the product $\mathbf{A}\mathbf{B}^{(1)}\mathbf{C}$ itself and some expressions involving it with respect to the choice of a generalized inverse $\mathbf{B}^{(1)}$ of \mathbf{B} , i.e., with respect to all matrices satisfying $\mathbf{B}\mathbf{B}^{(1)}\mathbf{B} = \mathbf{B}$. The collection of other expressions mentioned above contains in particular the range, rank, spectrum, trace, and Frobenius norm of $\mathbf{A}\mathbf{B}^{(1)}\mathbf{C}$. The purpose of the present paper is to revisit these results from the view-point of weakening the invariance requirement to the subsets of $\{\mathbf{B}^{(1)}\}$ comprising the matrices, which in addition to $\mathbf{B}\mathbf{B}^{(1)}\mathbf{B} = \mathbf{B}$ satisfy also further conditions from a definition of the Moore-Penrose inverse of \mathbf{B} , i.e., $\mathbf{B}^{(1)}\mathbf{B}\mathbf{B}^{(1)} = \mathbf{B}^{(1)}$ and/or $\mathbf{B}\mathbf{B}^{(1)} = (\mathbf{B}\mathbf{B}^{(1)})^*$ and/or $\mathbf{B}^{(1)}\mathbf{B} = (\mathbf{B}^{(1)}\mathbf{B})^*$.

Keywords

Generalized inverse, Range, Rank, Spectrum, Trace, Frobenius norm.