Characterizations of the commutativity of projectors referring to generalized inverses of their sum and difference

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Abstract

It is obvious that the commutativity of projectors (idempotent matrices) $\mathbf{P}_1$ and $\mathbf{P}_2$ is a sufficient condition for the products $\mathbf{P}_1\mathbf{P}_2$ and $\mathbf{P}_2\mathbf{P}_1$ to be projectors as well. The commutativity property becomes also a necessary condition when $\mathbf{P}_1$ and $\mathbf{P}_2$ are orthogonal projectors (Hermitian idempotent matrices). An extensive collection of results concerning both algebraic and statistical aspects of this property has been given by Baksalary (1987). Two of his algebraic results characterize the equality $\mathbf{P}_1\mathbf{P}_2 = \mathbf{P}_2\mathbf{P}_1$ of orthogonal projectors by referring to generalized inverses of the sum $\mathbf{P}_1 + \mathbf{P}_2$. The purpose of the present paper is to establish several generalizations of these results from two points of view: firstly, by relaxing the assumption of the orthogonality of projectors $\mathbf{P}_1$ and $\mathbf{P}_2$, and secondly, by considering also generalized inverses of the difference $\mathbf{P}_1 - \mathbf{P}_2$.

Keywords

Idempotent matrix, Hermitian idempotent matrix, Sum of projectors, Difference of projectors, Commutativity of projectors.

References:

Baksalary, J.K. (1987). Algebraic characterizations and statistical implications of the commutativity of orthogonal projectors, in: T. Pukkila, S. Puntanen (Eds.), *Proceedings of the Second International Tampere Conference in Statistics*, University of Tampere, Tampere, Finland, pp. 113-142.