

# Words in two positive definite letters

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## Abstract

A word in two letters  $\mathbf{A}$  and  $\mathbf{B}$  is symmetric if it reads the same right to left as left to right. We interpret juxtaposition as matrix multiplication and the two letters  $\mathbf{A}$  and  $\mathbf{B}$  are independent positive definite  $n$ -by- $n$  matrices. A symmetric word  $S(\mathbf{A}, \mathbf{B})$  is itself positive definite for any substitution of positive definite letters  $\mathbf{A}$  and  $\mathbf{B}$ . We call the equation  $S(\mathbf{A}, \mathbf{B}) = \mathbf{P}$  a symmetric word equation and we take one of the letters, say  $\mathbf{A}$  and the right hand side  $\mathbf{P}$  to be given positive definite matrices. The question then arises whether and how many positive definite solutions  $\mathbf{B}$  such an equation has. We also discuss methods for finding solutions to symmetric word equations and systems of symmetric word equations in more variables.