Mixing times and their application to perturbed Markov chains

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Abstract

A measure of the "mixing time" or "time to stationarity" in a finite irreducible discrete time Markov chain is considered. The statistic, $\eta_i = \sum_{j=1}^{m} m_{ij}\pi_j$, where $\{\pi_i\}$ is the stationary distribution and m_{ij} is the mean first passage time from state i to state j of the Markov chain, is shown to be independent of the state i that the chain starts in (so that $\eta_i = \eta$ for all i), is minimal in the case of a periodic chain, yet can be arbitrarily large in a variety of situations. An application concerning the affect perturbations of transition probabilities have on the stationary distributions of Markov chains leads to a new bound, involving η, for the 1-norm of the difference between the stationary probability vectors of the original and the perturbed chain. When η is large the stationary distribution of the Markov chain is very sensitive to perturbations of the transition probabilities.

Keywords

Markov chains, Stationary distribution, Mean first passage times, Mixing times, Perturbation theory, time to stationarity.