Blind identification of linear mixtures

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Abstract

The problem consists of identifying a $N \times P$ matrix, $\mathbf{A}$, from the sole (possibly noisy) observation of realizations of the N-dimensional random variable $\mathbf{x} = \mathbf{A}\mathbf{s}$, where $\mathbf{s}$ is an unknown non Gaussian P-dimensional random variable with independent components. Here, all variables take their values in the field of real or complex numbers. This paper addresses the case where $N < P$, which is referred to as the *under-determined* case, in certain communities. This problem is not new, but perhaps surprisingly, it has still not received a general answer of practical value, even if some identifiability results are available for a long time; e.g. see Kagan et al. (1973) and references therein. A summary of identifiability results is given. Then two practical approaches are proposed. The first makes use of the Pth derivatives of the joint second characteristic function of variables x_i. The main diffculty lies in the solution of a system of homogeneous polynomial equations of degree N in PN unknowns. A constructive numerical algorithm is proposed, and has been implemented for $2 \leq N \leq 4$ and $3 \leq P \leq 6$. The second approach makes use of the tensor of cumulants of order r of variable $\mathbf{x}$. Because of the observation model assumed, this tensor is structured, and it can be shown that matrix $\mathbf{A}$ can be recovered by exploiting the redundancies hidden in it, provided $r > 3$ and $2N \leq r(P-1)+2$. The algorithm again terminates within a finite number of steps, and has been implemented for both $r = 4$ and $r = 6$. This second family of algorithms can be seen to be related to the canonical decomposition of tensors, already addressed suboptimally in Kruskal (1977).

Keywords

Cumulants, Linear mixture, Characteristic function, Polynomial system, Deviation from Normality.

References:

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