A specific form of the generalized inverse of a partitioned matrix useful in econometrics

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Abstract

Faliva and Zoia (2002) provided an explicit formula for the inverse of the matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C}' & \mathbf{0} \end{pmatrix},$$

where $\mathbf{A} \in \mathbb{R}_{m\times m}$ and nonzero $\mathbf{B}, \mathbf{C} \in \mathbb{R}_{m\times n}$ are both of full column rank. They also showed the applicability of their result in econometrics. The additional assumption adopted by Faliva and Zoia is that the matrix $\mathbf{B}'_\perp \mathbf{A} \mathbf{C}_\perp$, with $\mathbf{B}_\perp$ and $\mathbf{C}_\perp$ spanning the orthogonal complements of the column spaces of $\mathbf{B}$ and $\mathbf{C}$, is of full rank. The purpose of the present paper is to generalize this formula from two points of view: firstly, by pointing out that the nonsingularity of $\mathbf{B}'_\perp \mathbf{A} \mathbf{C}_\perp$ is not necessary for the nonsingularity of $\mathbf{M}$ and, secondly, by removing the assumptions that $\mathbf{B}$ and $\mathbf{C}$ are of full column rank and considering the Moore-Penrose inverse of $\mathbf{M}$ instead of its usual inverse.

Keywords

Generalized inverse, Moore-Penrose inverse, Partitioned matrix.

References:

Faliva, M. and G. Zoia (2002). *Econometric Theory 18*, 525–530.