

# Further generalizations of a property of orthogonal projectors

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## Abstract

Generalizing the result in Lemma of Baksalary and Baksalary (2002), Baksalary, Baksalary, and Szulc (2002) have shown that if  $\mathbf{P}_1$  and  $\mathbf{P}_2$  are orthogonal projectors, then in all nontrivial cases a product of any length having  $\mathbf{P}_1$  and  $\mathbf{P}_2$  as its factors occurring alternately is equal to another such product if and only if  $\mathbf{P}_1$  and  $\mathbf{P}_2$  commute, in which case all products involving  $\mathbf{P}_1$  and  $\mathbf{P}_2$  reduce to the orthogonal projector  $\mathbf{P}_1\mathbf{P}_2$  ( $= \mathbf{P}_2\mathbf{P}_1$ ). In the present paper, we propose two further generalizations of this property. The first of them consists in replacing a product of the type described above, appearing on one of the sides of the equality under considerations, by an affine combination of two or three such products, while the second generalization is obtained by replacing the products appearing on the two sides of the original equation by linear combinations of two chain-products of  $\mathbf{P}_1$  and  $\mathbf{P}_2$ , where the scalars specifying these combinations have equal sums.

## Keywords

Hermitian idempotent matrix, Commutativity, Product of projectors.

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