

Relationships between partial orders of Hermitian matrices and their powers

Jerzy K. Baksalary

Zielona Góra University, Zielona Góra, Poland

Abstract

Baksalary and Pukelsheim (1991) considered the problem of how an order between two Hermitian nonnegative definite matrices \mathbf{A} and \mathbf{B} is related to the corresponding order between the squares \mathbf{A}^2 and \mathbf{B}^2 , in the sense of the star partial ordering, the minus partial ordering, and the Löwner partial ordering. Baksalary and Mitra (1991) and Groß (2001) developed characterizations of these orderings referring to the concept of the space preordering. Professor Ingram Olkin, who was the editor of the Baksalary and Pukelsheim's paper, asked several interesting questions concerning the results contained in it. Answers to some of them are presented here, showing possible generalizations of the above-mentioned results from two points of view: by relaxing the nonnegative definiteness assumption and by replacing the squares by arbitrary powers. The results obtained, accompanied by a set of comments, form a complete solution to the problem.

Keywords

Star partial ordering, Minus partial ordering, Löwner partial ordering, Space preordering, Nonnegative definite matrix, Hermitian matrix, Power of a matrix.

References:

- Baksalary, J.K. and S.K. Mitra (1991). Left-star and right-star partial orderings, *Linear Algebra Appl.* 149, 73-89.
- Baksalary, J.K. and F. Pukelsheim (1991). On the Löwner, minus, and star partial orderings of nonnegative definite matrices and their squares. *Linear Algebra Appl.* 151, 135-141.
- Groß, J. (2001). Löwner partial ordering and space preordering of Hermitian nonnegative definite matrices. *Linear Algebra Appl.* 326, 215-223.